

# Coupled Nonlinear Aeroelasticity and Flight Dynamics of Highly Flexible Aircraft

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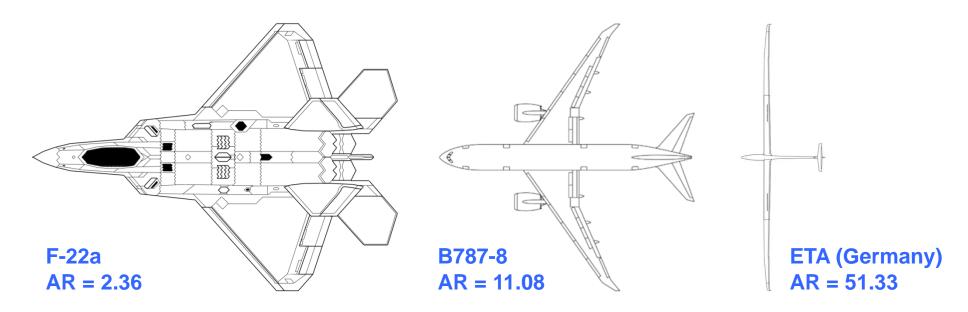
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#### Overview

- Introduction
  - Background
  - Motivation
- Theoretical formulation
  - Geometrically nonlinear beam
  - Unsteady aerodynamics
  - Flight dynamic modeling
- Numerical studies
- Concluding remarks
- Ongoing and future developments



# Aerodynamic Efficiency and Wing Aspect-Ratio



Large wing aspect-ratio to achieve high aerodynamic efficiency

$$R \text{ or } E \propto \left(\frac{L}{D}\right) \ln \left(\frac{W_{_{0}}}{W_{_{1}}}\right)$$



High aerodynamic efficiency



#### What about Structural Design?

- U.S. Air Force Sensorcraft studies
  - High-altitude, long-endurance
  - Unmanned vehicles
  - Sensor platform
  - Very high fuel fractions (up to 60%)

$$R \text{ or } E \propto \left(\frac{L}{D}\right) \ln \left(\frac{W_{_{0}}}{W_{_{1}}}\right)$$

- Very light structures
  - Not necessarily carry fuel, but...

Low structural weight fraction

**High-aspect**ratio wings











Highly flexible aircraft



### What's Challenging?

- Large wing deformation
  - Linear solution might not be sufficient
  - Nonlinear solution needed
- Coupling between wing oscillation and rigid-body motion



- Body freedom flutter
- Other effects
  - Low Reynolds flights
  - Local transonic effects







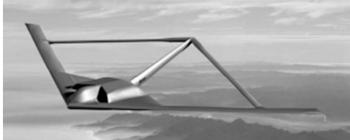
#### **Helios Solar Powered** Aircraft

Experiencing turbulence after taking off on first solar powered flight

July 14, 2001

Dryden Flight Research Center







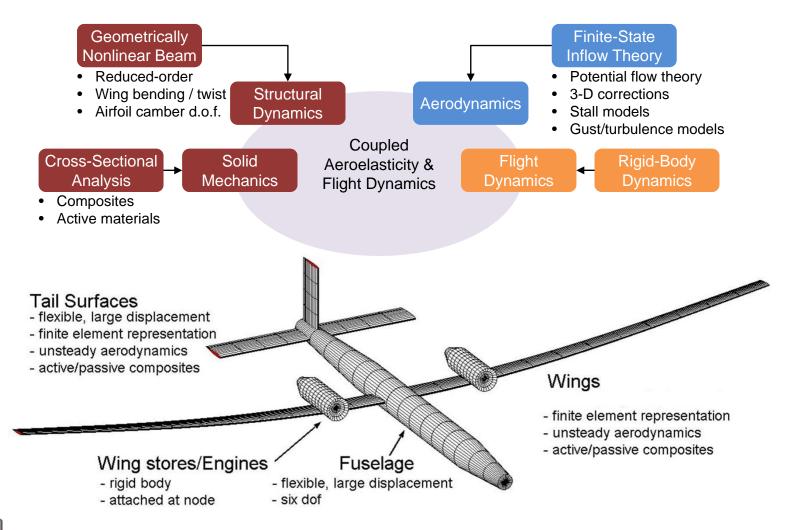
Need an integral solution for nonlinear aeroelasticity + flight dynamics

#### **Objectives**

- Create a low-order aeroelastic and flight dynamic framework
  - Effectively represent dynamic behavior of highly flexible vehicle
  - Efficient solution
  - Facilitate active aeroelastic tailoring and control studies
- Explore structural, aerodynamic, and control techniques to enhance flight efficiency and performance
  - Reduce drag
  - Reduce power consumption
  - Suppress instability
  - Reject air disturbance



# Coupled Nonlinear Aeroelasticity and Flight Dynamics

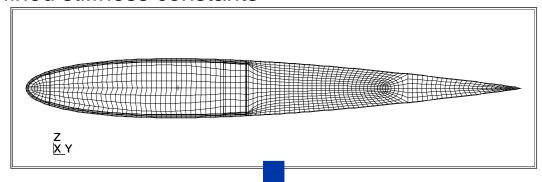




A simplified aeroelastic/flight dynamics simulation system

#### Reduced-Order Structural Modeling

- From 3D elastic problem to <u>2D beam cross-sectional analysis</u> and <u>1D beam model</u>
- Dimensional reduction using the Variational-Asymptotic Method:
  - Active thin-walled solution (mid-line discretization)
  - VABS (finite-element discretization)
  - User defined stiffness constants



Cross-Section Stiffness and Inertial Properties

$$\begin{bmatrix} F_{x} \\ M_{x} \\ M_{y} \\ M_{z} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{bmatrix} \mathcal{E}_{x} \\ K_{x} \\ K_{y} \\ K_{z} \end{bmatrix}$$



### **Basic Coordinate Systems**

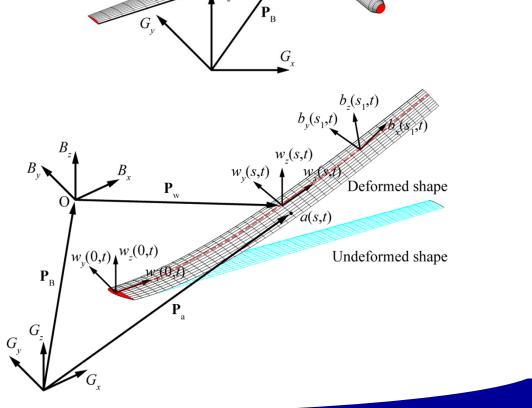
- Global frame (G)
- Body frame (B) origin not necessary to be C.G. of vehicle
- Body frame motion variables

$$b = \begin{cases} p_B \\ \theta_B \end{cases}$$

$$\dot{b} = \beta = \begin{cases} \dot{p}_{\scriptscriptstyle B} \\ \dot{\theta}_{\scriptscriptstyle B} \end{cases} = \begin{cases} v_{\scriptscriptstyle B} \\ \omega_{\scriptscriptstyle B} \end{cases}$$

$$\ddot{b} = \dot{\beta} = \left\{ \begin{array}{c} \ddot{p}_B \\ \ddot{\theta}_B \end{array} \right\} = \left\{ \begin{array}{c} \dot{v}_B \\ \dot{\omega}_B \end{array} \right\}$$

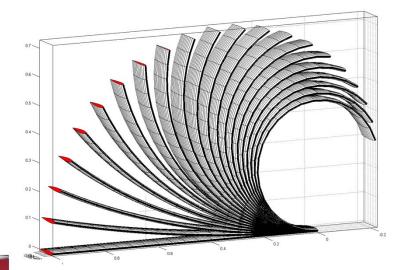
- Local beam frame (w)
- Auxiliary local frame (b)

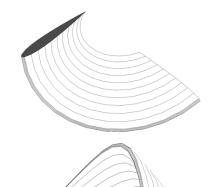




# Strained-Based Geometrically Nonlinear Beam Formulation

- Geometrically nonlinear beam formulation<sup>[1]</sup>
- Four local strain degrees-of-freedom ( $\varepsilon$ ): extension, twist, flatwise bending, and chordwise bending
- Constant-strain elements
- Capture large complex deformations with fewer elements computationally efficient
- Isotropic and anisotropic constitutive relations



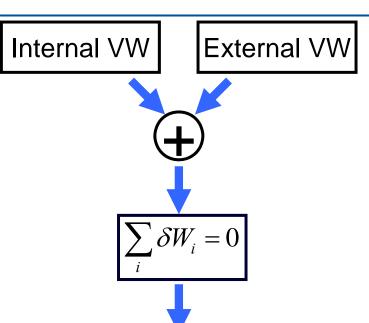


Sample element deformations with constant strain

Strains ( $\varepsilon$ ) and body velocities ( $\beta$ ) are independent variables

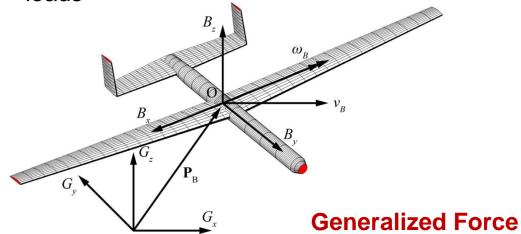
[1] Su, W., and Cesnik, C.E.S., "Strain-Based Geometrically Nonlinear Beam Formulation for Modeling Very Flexible Aircraft," *International Journal of Solids and Structures*, Vol. 48, No. 16-17, 2011, pp. 2349-2360. (doi: 10.1016/j.ijsolstr.2011.04.012)

# Formulation Based on Principle of Virtual Work



**Equations** of Motion

- Inertia force, internal strain, and strain rate
- Gravity loads, distributed loads, and point loads



$$\begin{bmatrix} M_{FF}(\varepsilon) & M_{FB}(\varepsilon) \\ M_{BF}(\varepsilon) & M_{BB}(\varepsilon) \end{bmatrix} \begin{bmatrix} \ddot{\varepsilon} \\ \dot{\beta} \end{bmatrix} + \begin{bmatrix} C_{FF}(\varepsilon, \dot{\varepsilon}, \beta) & C_{FB}(\varepsilon, \dot{\varepsilon}, \beta) \\ C_{BF}(\varepsilon, \dot{\varepsilon}, \beta) & C_{BB}(\varepsilon, \dot{\varepsilon}, \beta) \end{bmatrix} \begin{bmatrix} \dot{\varepsilon} \\ \beta \end{bmatrix} + \begin{bmatrix} K_{FF} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ b \end{bmatrix} = \begin{bmatrix} R_F \\ R_B \end{bmatrix}$$

#### **Generalized Mass**

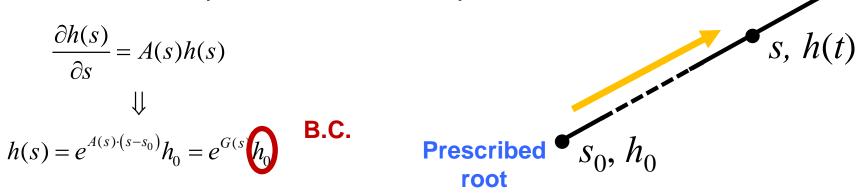
#### **Generalized Damping**

#### **Generalized Stiffness**

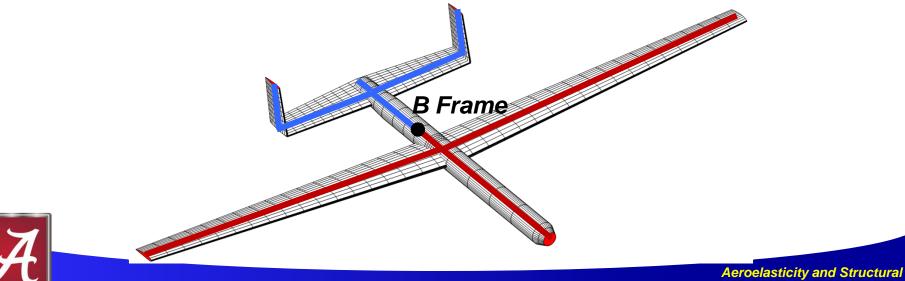
$$\begin{Bmatrix} R_F \\ R_B \end{Bmatrix} = \begin{Bmatrix} K_{FF} \varepsilon^0 \\ 0 \end{Bmatrix} - \begin{bmatrix} J_{h\varepsilon}^T \\ J_{hb}^T \end{bmatrix} Ng + \begin{bmatrix} J_{p\varepsilon}^T \\ J_{pb}^T \end{bmatrix} B^F F^{dist} + \begin{bmatrix} J_{\theta\varepsilon}^T \\ J_{\theta b}^T \end{bmatrix} B^M M^{dist} + \begin{bmatrix} J_{p\varepsilon}^T \\ J_{pb}^T \end{bmatrix} F^{pt} + \begin{bmatrix} J_{\theta\varepsilon}^T \\ J_{\theta b}^T \end{bmatrix} M^{pt}$$

### Recovery of Nodal Displacement

Solution of displacement-strain equation:



Marching kinematics in complete aircraft



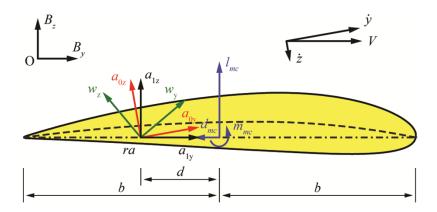
#### **Unsteady Aerodynamics**

• 2-D Theodorsen-like unsteady aerodynamics (Peters et al., 94, 95)

$$\begin{split} l_{mc} &= \pi \rho_{\infty} b^2 \left( -\ddot{z} + \dot{y} \dot{\alpha} - d \ddot{\alpha} \right) + 2\pi \rho_{\infty} b \dot{y}^2 \Bigg[ -\frac{\dot{z}}{\dot{y}} + \left( \frac{1}{2} b - d \right) \frac{\dot{\alpha}}{\dot{y}} - \bigodot{\dot{y}} \Bigg] + 2\pi \rho_{\infty} b c_1 \dot{y}^2 \delta \\ m_{mc} &= \pi \rho_{\infty} b^2 \left( -\frac{1}{8} b^2 \ddot{\alpha} - \dot{y} \dot{z} - d \dot{y} \dot{\alpha} - \dot{y} \bigodot{\partial} \right) + 2\pi \rho_{\infty} b^2 c_4 \dot{y}^2 \delta \end{split}$$

• Glauert expansion of inflow velocity as function of inflow states,  $\lambda_n$ 

$$\lambda_0 = \frac{1}{2} \sum_{n=1}^{N} b_n \lambda_n$$



• Finite state differential equation is transformed to independent variables  $\varepsilon$  and  $\beta$ 

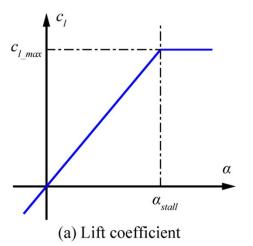
$$\dot{\lambda} = E_1 \lambda + E_2 \ddot{z} + E_3 \ddot{\alpha} + E_4 \dot{\alpha} \implies \left\{ \dot{\lambda} \right\} = F_1 \left\{ \begin{matrix} \ddot{\varepsilon} \\ \dot{\beta} \end{matrix} \right\} + F_2 \left\{ \begin{matrix} \dot{\varepsilon} \\ \beta \end{matrix} \right\} + F_3 \left\{ \lambda \right\}$$

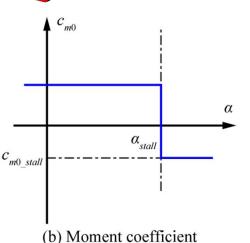
### Finite-State Inflow Theory: Modifications

- Aerodynamic coefficient modifications based on XFoil (Reeffects) or CFD calculations
- Compressibility accounted for by Prandtl-Glauert correction

 Spanwise aerodynamic corrections (3-D effects)

Simplified stall model



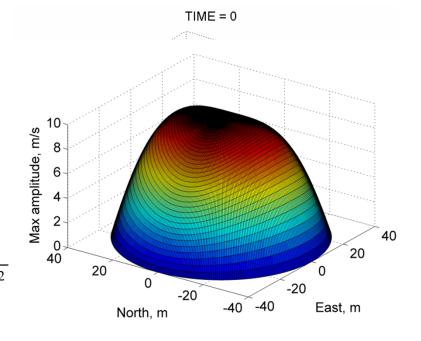




#### Discrete Non-uniform Gust Model

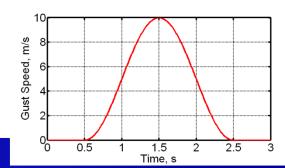
- Fixed region in space
- Amplitude distribution
  - Peak at center and zero at boundary
  - Possibly different distribution in East and North directions
  - Smooth transition

$$A(r, \eta, t) = \frac{1}{2} A_c \left[ 1 - \cos \left( 2\pi \frac{t}{t_g} \right) \right] \sqrt{\left( A_E \cos \eta \right)^2 + \left( A_N \sin \eta \right)^2}$$



$$A_{E}(r) = \sin\left(\frac{\pi}{2}\left[1 - \left(\frac{r}{r_{0}}\right)^{n_{E}}\right]\right) , \quad A_{N}(r) = \sin\left(\frac{\pi}{2}\left[1 - \left(\frac{r}{r_{0}}\right)^{n_{N}}\right]\right) , \quad 0 < r \le r_{0}$$

Time variation: 1-cosine with different temporal durations



#### Dryden Gust Model

#### Gust PSD function

$$\Phi_{w}(\omega_{m}) = \frac{\sigma_{w}^{2}L_{w}\left[1 + 3\left(\frac{L_{w}\omega_{m}}{U_{0}}\right)^{2}\right]}{\pi U_{0}\left[1 + \left(\frac{L_{w}\omega_{m}}{U_{0}}\right)^{2}\right]^{2}}$$

$$\Phi_{w}(\omega_{m}) = \frac{2\sigma_{w}^{2}L_{w}\left[1+12\left(\frac{L_{w}\omega_{m}}{U_{0}}\right)^{2}\right]}{\pi U_{0}\left[1+4\left(\frac{L_{w}\omega_{m}}{U_{0}}\right)^{2}\right]^{2}}$$

MIL-HDBK-1797

- $\omega_m$ : Frequency component (rad/s)
- $U_0$ : Free stream velocity (m/s)
- $L_w$ : Scale of turbulence (m), determined by altitude (m)
- Superposition of all frequency components with random phase

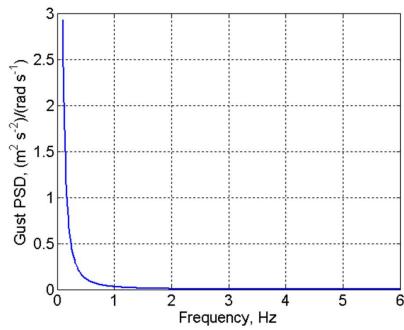


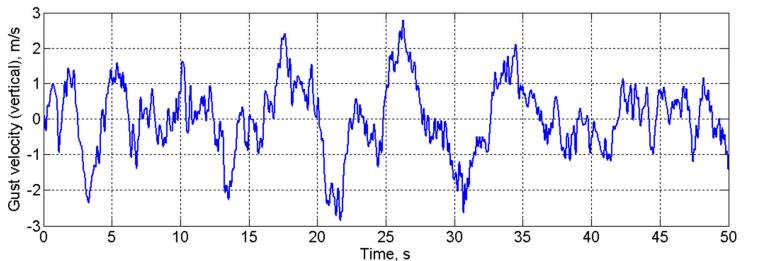
$$w(t) = \sum_{m=1}^{\infty} \sqrt{\Phi_w(\omega_m) \Delta \omega} \cos(\omega_m t + \psi_m)$$

# PSD and Time History of Gust Velocity

- Frequency band [0.1~6] Hz
- $\sigma_w$  adjusted to obtain enough wing deformation
- Uniform spanwise distribution

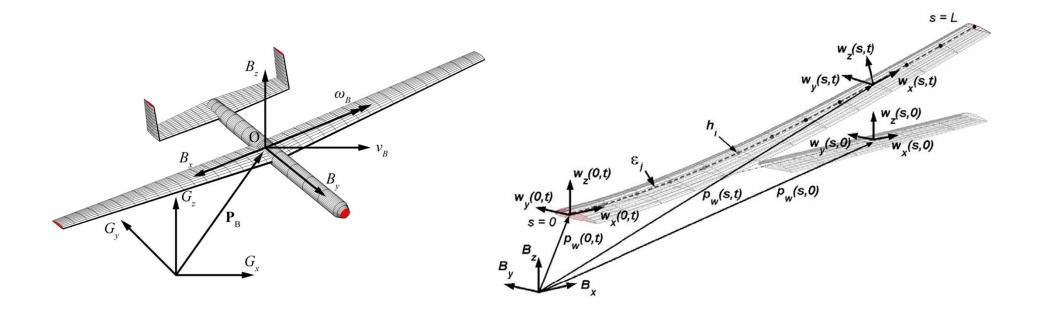
# Power concentrated at the low frequency range





### Flight Dynamics Modeling

The trajectory and orientation of a fixed body reference frame, *B*, at point *O*, which in general is *not* the aircraft's center of mass





#### Full Air Vehicle Model for Flight Simulations

Elastic equations of motion

→ Strains (4 by m structural d.o.f.)

$$M(\varepsilon) \begin{cases} \ddot{\varepsilon} \\ \dot{\beta} \end{cases} + C(\varepsilon, \dot{\varepsilon}, \beta) \begin{cases} \dot{\varepsilon} \\ \beta \end{cases} + K \begin{cases} \varepsilon \\ b \end{cases} = R(\varepsilon, \dot{\varepsilon}, \ddot{\varepsilon}, \zeta, \beta, \dot{\beta}, \lambda )$$
Control inputs
Body velocities (6 flight dynamic d.o.f.)

Finite-state 2-D unsteady aerodynamics

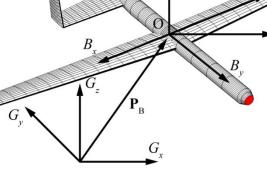
Inflow states (N by m aerodynamic d.o.f.)

$$\dot{\lambda} = F_1 \left\{ \frac{\ddot{\varepsilon}}{\dot{\beta}} \right\} + F_2 \left\{ \frac{\dot{\varepsilon}}{\beta} \right\} + F_2 \lambda$$



$$\dot{\zeta} = -\frac{1}{2}\Omega$$
Frame orientation (4 quaternions)





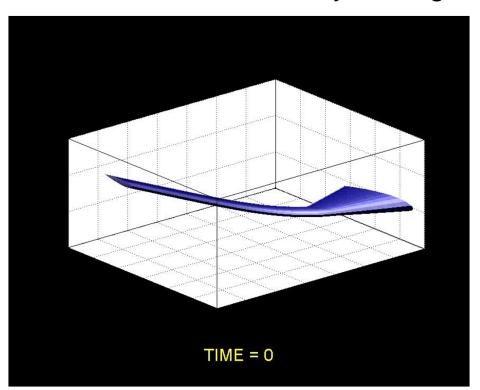
Inertial velocities (6 d.o.f.)

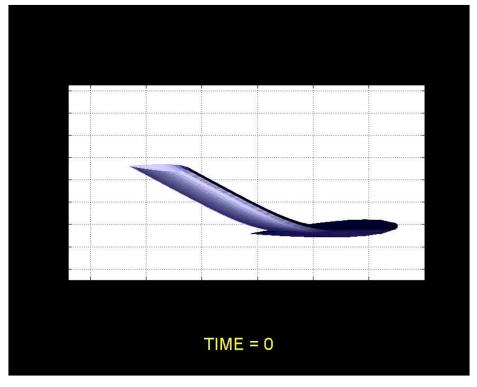
#### **Numerical Studies**



#### Flutter of Constrained Vehicle

- Similar to constrained wind-tunnel model (no body DOFs)
- Fixed root angle of attack (8 deg)
- Free stream velocity 1% higher than flutter speed





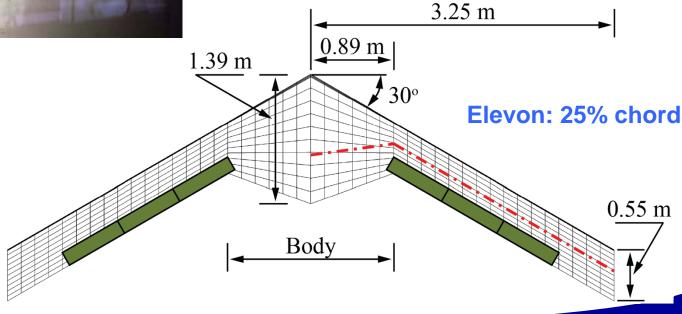


Coupled out-of-plane bending/torsion/in-plane bending mode

# Blended-Wing-Body (BWB) Model

Properties inspired from HiLDA (High Lift over Drag Active Wing) wind-tunnel model





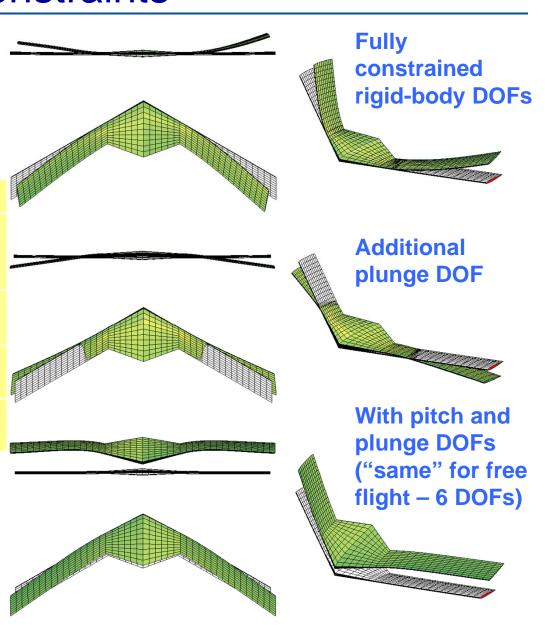


# Comparison of Flutter Modes with Rigid-Body Constraints

All cases trimmed for 6,096 m (20,000 ft) altitude, same fuel condition

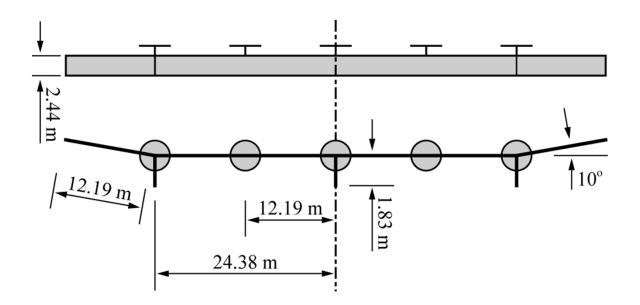
	Flutter Speed	Frequency
Fully constrained dof's	172.52 m/s	7.30 Hz
+ plunging	164.17 m/s	7.07 Hz
+ pitching and plunging	123.17 m/s	3.32 Hz
Free flight	123.20 m/s	3.32 Hz

Traditional wind-tunnel setup
maybe non-conservative – need
rigid-body DOFs in the aeroelastic
analyses, simulations, and tests



## Highly Flexible Flying Wing Model

- Representative of Helios prototype<sup>[2]</sup>
  - Five engines and three pods
  - Payloads applied at center pod
  - Empty gross mass: 726 kg

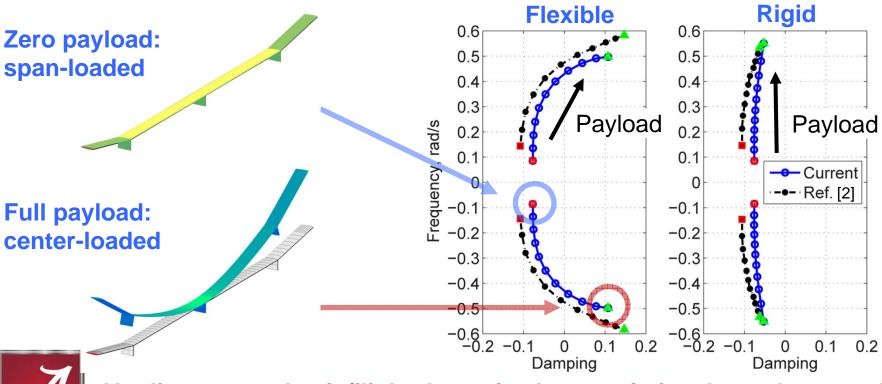




[2] Patil, M.J., and Hodges, D.H., "Flight Dynamics of Highly Flexible Flying Wings," *Journal of Aircraft*, Vol. 43, No. 6, 2006, pp. 1790-1798.

#### Trim Results and Flight Stability

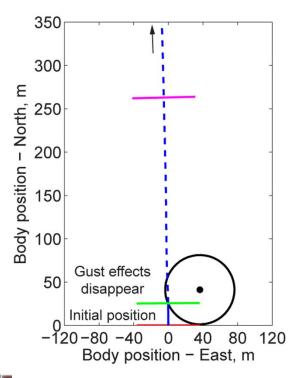
- Speed: 12.2 m/s at sea level; Payload: 0 227 kg (at center pod)
- Linearization about each trimmed condition with increase of payloads
- Root locus for phugoid mode (left: flexible, right: rigid)
- Unstable phugoid mode for payload > 152 kg

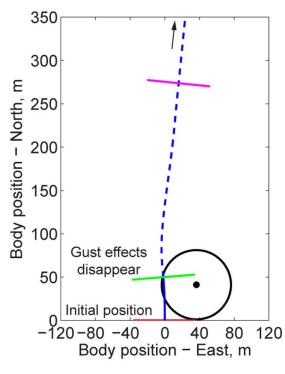


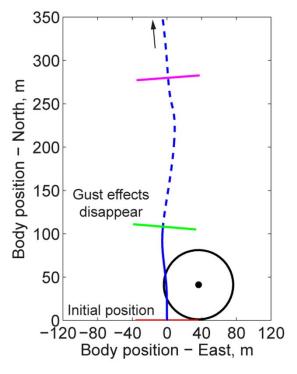
Nonlinear aeroelastic/flight dynamic characteristics dependent on trim conditions

# Non-symmetric Gust Input and Response – Fully-Loaded Configuration

- Payload: 227 kg; gust region radius: 40 m; maximum gust center amplitude: 10 m/s
- Non-symmetric discrete gust distribution:
  - gusts mainly applied on right wing







2 s gust duration

4 s gust duration

8 s gust duration

Gust duration impacts after-gust flight path

# Instantaneous Vehicle Positions and Orientations

 Positions and orientations at 0, 5, 12, 18, 24, and 30 s, respectively



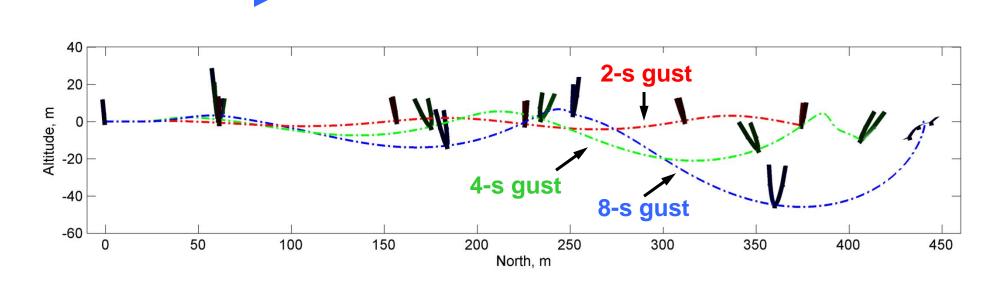
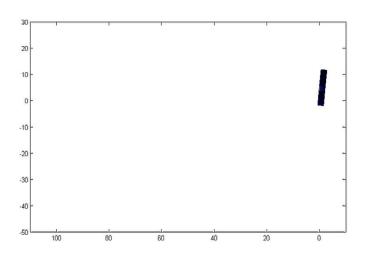
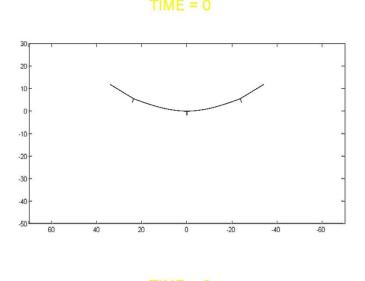




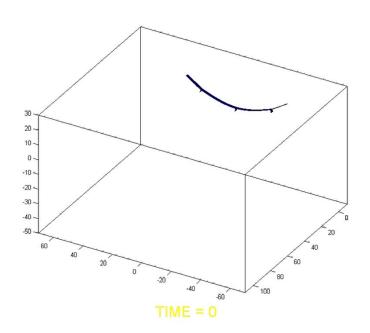
Illustration of unstable Phugoid mode

# Animation of Vehicle Motion with Gust Perturbations





2-s gust 4-s gust 8-s gust



### **Concluding Remarks**

- Framework for modeling and analyzing highly flexible aircraft
  - Coupled nonlinear aeroelastic/flight dynamic simulation
  - Strain-based geometrically-nonlinear beam
  - Incompressible unsteady aerodynamics (with compressibility corrections and stall models)
  - Rigid-body flight dynamics
- Highly flexible aircraft have radically different behavior than conventional aircraft
  - Coupling between aircraft deformation and rigid-body motions changes flutter boundaries
  - Flutter boundary in free flight condition may be different from constrained flight
  - Finite amplitude gust can excite instabilities



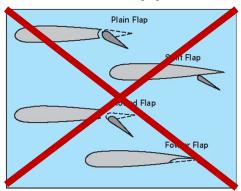
### Concluding Remarks (Cont'd)

- What did we learn from the physics of highly flexible aircraft?
  - Operating (trim) condition should be the basis in weight, structural, and stability analyses
    - Deformed geometry other than the undeformed shape
  - Traditional linear solution to highly flexible aircraft aeroelasticity might not be sufficient
    - Nonlinear solution is required
  - Coupling between aeroelasticity and flight dynamics needs to be considered
    - Aeroelastic models should incorporate the rigid-body motion, and vice versa
    - Individual solutions might not be appropriate

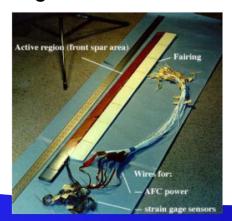


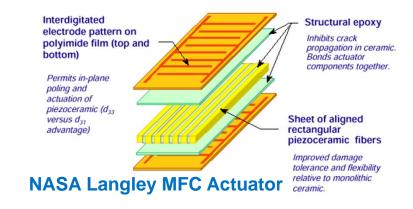
#### Active Aeroelastic Tailoring and Control

Traditional approach for aerodynamic/flight control

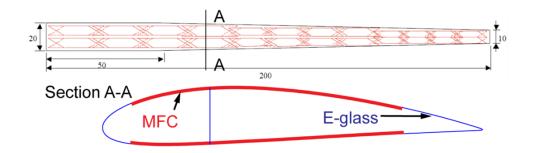


- Drag due to control surfaces
- Conformal wing shape changes
  - Integral strain actuation of bending/twist





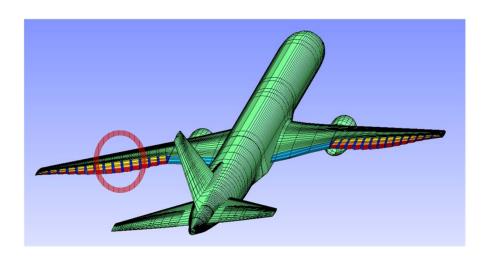


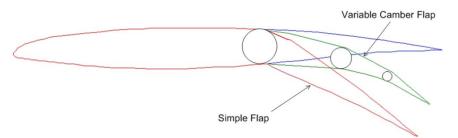




### Wing Camber Change

NASA VCCTEF





- Jointly proposed by UA/GA Tech/OSU/MSU
  - Full variable camber wing

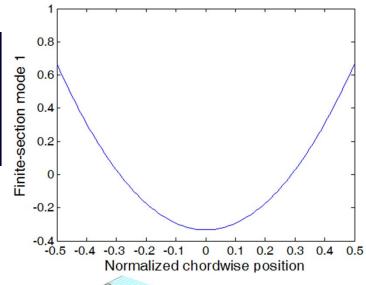


#### Recent Development

- Wing cross-sectional warping
  - Plate-like modeling capability with beam model
  - Augmented EoM with camber degrees (finite-section modes)

$$M(\varepsilon) \begin{Bmatrix} \ddot{\varepsilon} \\ (\ddot{q}_n) \\ \dot{\beta} \end{Bmatrix} + C(\varepsilon, \dot{\varepsilon}, \beta) \begin{Bmatrix} \dot{\varepsilon} \\ (\dot{q}_n) \\ \beta \end{Bmatrix} + K \begin{Bmatrix} \varepsilon \\ (q_n) \\ b \end{Bmatrix} = \begin{Bmatrix} R_{\varepsilon} \\ R_{q_n} \\ R_{b} \end{Bmatrix}$$

 Impact on aeroelasticity, flight dynamics, and control -> on-going



Shell

S-Beam, no Warping

S-Beam, warping



#### **Linear Strain Modes**

Approximate solutions using strain modes

$$\varepsilon(s,t) = \Phi(s)\eta(t)$$

Modes from elastic EOM

$$\begin{bmatrix} M_{FF} & M_{FB} \\ M_{BF} & M_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{\mathcal{E}} \\ \dot{\beta} \end{Bmatrix} + \begin{bmatrix} K_{FF} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \mathcal{E} \\ b \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Phi_{C} = \begin{Bmatrix} \Phi_{F} \\ \Phi_{B} \end{Bmatrix}$$

Only take the elastic components of the modes

$$\Phi(s) = \Phi_F$$



#### **Modal Equations**

$$\begin{split} M_{FB}\ddot{\mathcal{E}} + M_{FB}\dot{\beta} + C_{FB}\dot{\mathcal{E}} + C_{FB}\beta + K_{FB}\mathcal{E} &= R_F \\ M_{BB}\ddot{\mathcal{E}} + M_{BB}\dot{\beta} + C_{BB}\dot{\mathcal{E}} + C_{BB}\beta &= R_B \\ \dot{\lambda} &= F_1 \left\{ \begin{matrix} \dot{\mathcal{E}} \\ \dot{\beta} \end{matrix} \right\} + F_2 \left\{ \begin{matrix} \dot{\mathcal{E}} \\ \dot{\beta} \end{matrix} \right\} + F_3\lambda \end{split}$$



$$\begin{split} \overline{M}_{FF}\ddot{\eta} + \overline{M}_{FB}\dot{\beta} + \overline{C}_{FF}\dot{\eta} + \overline{C}_{FB}\beta + \overline{K}_{FF}\eta &= \overline{R}_F(\eta,\dot{\eta},\ddot{\eta},\beta,\dot{\beta}) \\ \overline{M}_{BF}\ddot{\eta} + \overline{M}_{BB}\dot{\beta} + \overline{C}_{BF}\dot{\eta} + \overline{C}_{BB}\beta &= \overline{R}_B(\eta,\dot{\eta},\ddot{\eta},\beta,\dot{\beta}) \\ \dot{\lambda} &= \left[\overline{F}_{1F} \quad F_{1B}\right] \left\{\begin{matrix} \dot{\eta} \\ \dot{\beta} \end{matrix}\right\} + \left[\overline{F}_{2F} \quad F_{2B}\right] \left\{\begin{matrix} \dot{\eta} \\ \dot{\beta} \end{matrix}\right\} + F_3\lambda \end{split}$$



### Highly Flexible Wing

#### Beam properties:

Length (m)	16	16
Chord (m)	1	14
Mass per length (kg/m)	0.75	12
x-sectional c.g. position	50% chord	10
x-sectional shear center	50% chord	
Rotational inertia (kg·m)	0.1	· ·
Flat bending rigidity (N·m²)	$2.00 \times 10^4$	•
Edge bending rigidity (N·m <sup>2</sup> )	$4.00 \times 10^4$	869 2
Torsional rigidity (N·m²)	$1.00 \times 10^4$	° 62 9,34 0

#### Nonlinear flutter speed: 23 m/s

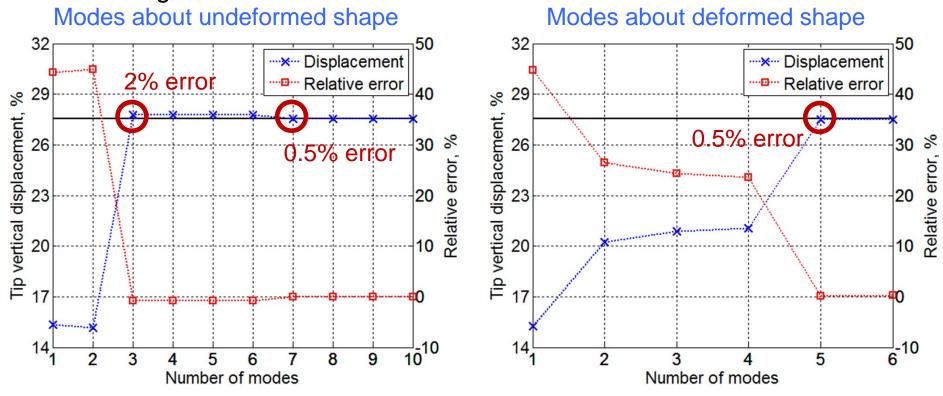
	Ref. 3*	Current (linear)	Current (nonlinear)
Velocity (m/s)	32.2	32.2	23.3
Frequency (Hz)	3.60	3.60	1.61



[3] Patil, M.J., Hodges, D.H. and Cesnik, C.E.S., "Nonlinear Aeroelasticity and Flight Dynamics of High-Altitude Long-Endurance Aircraft," *Journal of Aircraft*, Vol. 38, No. 1, 2001, pp. 88-94.

#### **Modal-Based Static Solution**

Convergence of static solutions with different number of modes

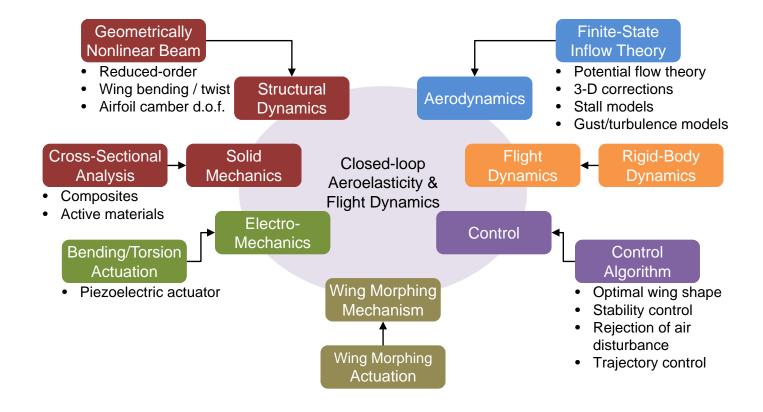


For more discussion:

[4] Su, W., and Cesnik, C.E.S., "Strain-Based Analysis for Geometrically Nonlinear Beams: a Modal Approach," *Journal of Aircraft*, Vol. 51, No. 3, 2014, pp. 890–903. (doi: 10.2514/1.C032477)

Fewer modes required if modes are obtained about deformed shape

#### Multi-disciplinary Simulation of Flight Vehicles





An active aero-servo-elastic simulation system

